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RESILIENT CONTROL OF LINEAR TIME-INVARIANT NETWORKED MULTIAGENT SYSTEMS

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Abstract

A local state emulator-based adaptive control law is proposed for multiagent systems with agents having linear time-invariant dynamics. Specifically, we present and analyze a distributed adaptive control architecture, where agents achieve system-level goals in the presence of exogenous disturbances. Apart from existing relevant literature that makes specific assumptions on network topologies, agent dynamics, and/or the fraction of agents subjected to disturbances, the proposed approach allows agents to achieve system-level goals — even when all agents are subject to exogenous disturbances. Several numerical examples are provided to demonstrate the efficacy of our approach.

1 Introduction

The distributed control of networked multiagent systems, in which groups of agents work together to achieve a common goal through local peer-to-peer information exchange, has seen many advancements in the past decade (see, for example, [1], [2], and references therein). Such networks are envisioned for applications in demanding, human interactive, and safety critical systems where resilience in the presence of disturbances is required. Until recently, however, much work has focused on fixed-gain distributed controllers, which are unable to recover the desired performance in the presence of unknown exogenous disturbances as outlined in [3] and [4]. Specifically, these systems do not have a centralized mechanism to monitor for node failures, malicious attacks, network link failures, and other disturbances, which can lead to system instability and failure to achieve the system-level goals as described in [3] and [5].

Several approaches, most notably in [6–9], have been developed to detect node disturbances and mitigate their effects. These approaches simply assume that a node's information is no longer usable and all information from the node is ignored, which may not be appropriate in scenarios where the effect of the disturbance can be suppressed. The authors of [8] and [10] make assumptions on the network topology (other than the standard assumption of connectedness) requiring the underlying communication network to be known. In addition, [7], [10], and [11] assume that a maximum number of nodes are disturbed, which can be a strict assumption in hostile environments. Computationally expensive observer techniques are considered in [8] and [9]. In [12], the authors focus on discovering subsets of disturbed nodes and require neighboring nodes to mitigate the disturbance effects.

To address the short–comings of current approaches, we propose in this paper a distributed adaptive control approach for a benchmark consensus problem, without loss of generality, in the presence of exogenous disturbances for agents with linear time-invariant dynamics. Specifically, in order to achieve the desired network performance, an adaptive control approach utilizing local state emulators is employed. Similar approaches are reported in [13] and [14], but only for the case where agents have single integrator dynamics. While the authors of [15–17] consider the consensus problem for agents with disturbed linear time-invariant dynamics, [15] only considers disturbances which are polytopic in nature, [16] considers that agents must track an undisturbed leader, which is not practical for leader–less networks, and [17] requires that agents exchange their disturbance estimates as well as their state estimates, which incurs a higher communication cost, and which assumes that the communication channels are not disturbed. We show that for agents with linear time invariant dynamics, the effects of exogenous disturbances affecting any subset (or all) agents can be mitigated though local state information exchange.

The organization of this paper is as follows. First, we introduce some necessary notation from linear algebra and graph theory used throughout this paper. We then present the main results of this paper where we demonstrate the stability of the system in the presence of constant disturbances. Finally, we demonstrate the efficacy of our proposed control approach with several numerical examples.

2 Mathematical Preliminaries

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}^n$ denotes the set of real $n\times 1$ column vectors, $\mathbb{R}^{m\times n}$ denotes the set of real $m\times n$ matrices, $\mathbb{R}_+$ denotes a set of positive real numbers, $\mathbb{R}_+^{m\times n}$ (resp., $\bar{\mathbb{R}}_+^{m\times n}$) denotes a set of real $m\times n$ positive definite (resp., nonnegative-definite) real matrices, $\mathbb{S}_+^{m\times n}$ (resp., $\bar{\mathbb{S}}_+^{m\times n}$) denotes a set of real, positive definite (resp., nonnegative definite) symmetric real matrices, $\mathbb{Z}$ the set of integers, $\mathbb{Z}_+$ (resp., $\bar{\mathbb{Z}}_+$) denotes the set of positive (resp., nonnegative) integers, $\mathbf{0}_n$ an $n\times 1$ vector with 0 entries, $\mathbf{1}_n$ an $n\times 1$ vector with all entries set to 1, $\mathbf{0}_{m\times n}$ a $m\times n$ matrix with all entries set to 0, $\mathbf{1}_{m\times n}$ a $m\times n$ matrix with all entries set to 1, and I_n denotes the $n\times n$ identity matrix. Furthermore, we write $(\cdot)^T$ for the transpose, $\|\cdot\|^2$ for the Euclidean norm, $\lambda_i(A)$ for the i-th eigenvalue of A (ordered from least to greatest), $\mathrm{diag}(a)$ for the diagonal matrix with the vector a on its diagonal, $[A]_{ij}$ for the entry of the matrix A on the i-th row and j-th column, $\mathrm{spec}(A)$ for the ordered spectrum of the matrix A, and J_A for the Jordan decomposition of the matrix A.

Next, we recall some of the basic notions from graph theory, where we refer to references [4] and [18] for further details. An *undirected* graph $\mathcal{G}$ is defined by a set $\mathcal{V}_\mathcal{G}=\{1,\ldots,N\}$ of *nodes* and a set $\mathcal{E}_\mathcal{G}\subset\mathcal{V}_\mathcal{G}\times\mathcal{V}_\mathcal{G}$ of *edges*. Furthermore, the number of agents, N, in the network is given by $N=|\mathcal{V}_\mathcal{G}|$. If $(i,j)\in\mathcal{E}_\mathcal{G}$, then the nodes i and j are *neighbors* and the neighboring relation is indicated with $i\sim j$. The *degree* of a node is given by the number of its neighbors. Letting d_i be the degree of node i, then the *degree* matrix of a graph $\mathcal{G}$, $\mathcal{D}(\mathcal{G})\in\mathbb{R}^{n\times n}$, is given by $\mathcal{D}(\mathcal{G})\triangleq\mathrm{diag}(d)$, $d=[d_1,\ldots,d_N]^{\mathrm{T}}$. A *path* $i_0i_1\ldots i_L$ is a finite sequence of nodes such that $i_{k-1}\sim i_k$, $k=1,\ldots,L$, and a graph $\mathcal{G}$ is *connected* if there is a path between any pair of distinct nodes. The *adjacency* matrix of a graph $\mathcal{G}$, $\mathcal{A}(\mathcal{G})\in\mathbb{R}^{n\times n}$, is given by

$$[\mathcal{A}(\mathcal{G})]_{ij}\triangleq\begin{cases}1, & \text{if } (i,j)\in\mathcal{E}_\mathcal{G},\\ 0, & \text{otherwise.}\end{cases}$$

The *Laplacian* matrix of a graph, $\mathcal{L}(\mathcal{G})\in\bar{\mathbb{S}}_+^{n\times n}$, playing a central role in many graph theoretic treatments of multiagent systems, is given by $\mathcal{L}(\mathcal{G})\triangleq\mathcal{D}(\mathcal{G})-\mathcal{A}(\mathcal{G})$. The spectrum of the Laplacian of a connected, undirected graph can be ordered as

$$0=\lambda_1(\mathcal{L}(\mathcal{G}))<\lambda_2(\mathcal{L}(\mathcal{G}))\leq\cdots\leq\lambda_n(\mathcal{L}(\mathcal{G})). \tag{1}$$

Furthermore, there exist $p,\ q\in\mathbb{R}^n$ such that

$$q^{\mathrm{T}}\mathcal{L}(\mathcal{G})=0,\quad \mathcal{L}(\mathcal{G})p=0, \tag{2}$$

and $q^{\mathrm{T}}p=1$. Note that q and p are normalized left and right eigenvectors associated with the zero eigenvalue of $\mathcal{L}(\mathcal{G})$, respectively. For the ease of exposition, we will assume $p=\mathbf{1}_N$ for the reminder of this paper without loss of generality. Throughout this paper, we model a given multiagent system by a connected, undirected graph $\mathcal{G}$, where nodes and edges represent agents and inter-agent communication links, respectively. Finally, the results of the following lemma will be used through this paper.

Lemma 1 (Theorem 2, [19]). Consider a group of agents communicating over a connected, undirected graph $\mathcal{G}$ where each agent has local dynamics given by

$$\dot{x}_i(t)=Ax_i(t)+Bu_i(t),\quad x_i(t_0)=x_{i0}, \tag{3}$$

$$y_i(t)=Cx_i(t), \tag{4}$$

subject to the controller

$$u_i(t)=-\left[K\sum_{i\sim j}(y_i(t)-y_j(t))\right]. \tag{5}$$

If

$$\mathrm{Rank}(C)=\mathrm{Rank}\begin{pmatrix}C\\ B^{\mathrm{T}}\bar{P}\end{pmatrix}, \tag{6}$$

where $\bar{P}\in\mathbb{S}_+^{n\times n}$ satisfies

$$\bar{P}A+A^{\mathrm{T}}\bar{P}-2\bar{P}BB^{\mathrm{T}}\bar{P}+\mathrm{I}_n=0, \tag{7}$$

then, if the feedback gain matrix $K=\max\{1,\lambda_{k,min}^{-1}\}K_0$, where $\lambda_{k,min}$ is the minimum non-zero eigenvalue (Fielder eigenvalue) of the associated Laplacian, and K_0 is a solution of

$$K_0C=B^{\mathrm{T}}\bar{P}, \tag{8}$$

the eignevalues of

$$A-\lambda_k(\mathcal{L}(\mathcal{G}))BKC,\quad k>1, \tag{9}$$

$$A-d_iBKC,\quad i\in\mathcal{V}(\mathcal{G}), \tag{10}$$

lie in the open left half plane, that is $\mathrm{spec}(A-\lambda_k(\mathcal{L}(\mathcal{G}))BKC)<0$, and $\mathrm{spec}(A-d_iBKC)<0$, where $\lambda_k(\mathcal{L}(\mathcal{G}))$ are the non-zero eigenvalues of the associated Laplacian matrix.

3 Resilient Networks for Linear Time-Invariant Systems

In this section, we propose a networked control approach for coordination of agents with linear time-invariant dynamics in the presence of persistent local agent disturbances. First, we present the local agent system dynamics and propose a novel method for mitigating the effects of local agent disturbances. Next, we demonstrate that the agent dynamics converge to the designed local state emulator dynamics. Finally, we characterize and rigorously analyze the performance of the local agent state emulator.

3.1 Problem Formulation

Consider a networked multiagent system whose agents are subject to disturbances such that their dynamics are given by

$$\dot{x}_i(t) = Ax_i(t) + B(u_i(t) + w_i),\ x_i(t_0) = x_{i0}, \tag{11}$$

$$y_i(t) = Cx_i(t), \tag{12}$$

where $x_i(t) \in \mathbb{R}^n$ denotes the state of agent $i, i = 1,2,\dots,N$, $u_i(t) \in \mathbb{R}^m$ denotes the control input to agent i, $w_i \in \mathbb{R}^m$ denotes the constant unknown disturbance affecting agent i, $y_i(t) \in \mathbb{R}^l$ denotes the output of agent i, $A \in \mathbb{R}^{n\times n}$ denotes the local agent state transition matrix of agent i, $B \in \mathbb{R}^{n\times m}$ denotes the control input matrix of agent i, $C \in \mathbb{R}^{l\times n}$ denotes the output matrix of agent i, and we assume the system (A,B,C) is stabilizable and detectable.

Remark 1. Note that a local controller may be used to place the eigenvalues of the local state transition matrix A to achieve a desired response.

Since our aim is to mitigate the effect of local disturbances in order to synchronize agent outputs, consider the relative output feedback controller

$$u_i(t) = -\left[K\sum_{i\sim j}(y_i(t) - y_j(t))\right] - \hat{w}_i(t), \tag{13}$$

where $K \in \mathbb{R}^{m\times m}$ is an output feedback gain matrix, $y_j(t)$ is the output of agent $j, j = 1,2,\dots,N$, and $\hat{w}_i(t)$ is the estimate of the disturbance of agent i to be designed. Using (12) and (13) in (11), the local agent dynamics can be rewritten as

$$\begin{aligned}\dot{x}_i(t) &= Ax_i(t) - B\left[K\sum_{i\sim j}(Cx_i(t) - Cx_j(t)) + \hat{w}_i(t) - w_i\right],\\ &= Ax_i(t) - B\left[KC\sum_{i\sim j}(x_i(t) - x_j(t)) + \hat{w}_i(t) - w_i\right].\end{aligned} \tag{14}$$

Next, consider the local state emulator for agent i, characterizing the *desired* local behavior, given by

$$\dot{\hat{x}}_i(t) = A\hat{x}_i(t) - BKC\sum_{i\sim j}(\hat{x}_i(t) - x_j(t)), \quad \hat{x}_i(t_0) = x_{i0}, \tag{15}$$

where $\hat{x}_i(t)$ denotes the state emulator state of agent i, and note that while the state emulator has no local disturbance sources, disturbances may enter through information exchange.

Our next objective is to design a local weight update law $\hat{w}_i(t)$ to mitigate the effect of the local disturbance w_i. To this end, consider the weight update law given by

$$\dot{\hat{w}}_i(t) = \alpha B^{\mathrm{T}} P_i \tilde{x}_i(t), \quad w_i(t_0) = w_{i0}, \tag{16}$$

with $\alpha > 0$ being the system learning rate, $\tilde{x}_i(t)$ denoting the system error defined as

$$\tilde{x}_i(t) \triangleq x_i(t) - \hat{x}_i(t), \tag{17}$$

and $P_i \in \mathbb{S}_+^{n\times n}$ satisfies the Lyapunov equation

$$P_i(A - d_iBKC) + (A - d_iBKC)^{\mathrm{T}}P_i + Q_i = 0, \tag{18}$$

where $Q_i \in \mathbb{S}_+^{n\times n}$.

Remark 2. If K is chosen according to (8), then spec$(A - d_iBKC) < 0$ as a direct result of Lemma 1, which implies solutions to (18) exist. Note that Lemma 1 only gives sufficient conditions for solutions to exist, and there may be other methods to choose a valid stabilizing matrix.

Now, using (14) and (15), the system error dynamics, characterizing the difference between the local agent dynamics and the desired local state emulator dynamics, can be given by

$$\dot{\tilde{x}}_i = (A - d_iBKC)\tilde{x}_i(t) + B\widetilde{w}_i(t),\ \tilde{x}_i(t_0) = 0, \tag{19}$$

where the weight update error is defined as

$$\widetilde{w}_i(t) \triangleq w_i - \hat{w}_i(t), \tag{20}$$

and

$$\dot{\widetilde{w}}_i(t) = -\dot{\hat{w}}_i(t). \tag{21}$$

Finally, using (19), the local agent state emulator in (15) can be rewritten as

$$\begin{aligned}\dot{\hat{x}}_i(t) &= A\hat{x}_i(t) - BKC\sum_{i\sim j}(\hat{x}_i(t) - x_j(t)) \pm BKC\sum_{i\sim j}\hat{x}_j(t),\\ &= A\hat{x}_i(t) - BKC\sum_{i\sim j}(\hat{x}_i(t) - \hat{x}_j(t)) + BKC\sum_{i\sim j}\tilde{x}_j(t).\end{aligned} \tag{22}$$

This concludes the setup of our problem. In the next section, we present the performance and stability guarantees for the system given by (11) and (12) subject to the controller (13).

3.2 Performance and Stability Analysis of the Closed-Loop Error Dynamics

In this section, we begin our analysis of the networked multiagent system. Specifically, we give sufficient conditions to demonstrate that the local agent dynamics converge to the desired state emulator dynamics. Note that we will discuss the stability and performance of the state emulator dynamics in the next section.

In the next theorem, we show that the system state $x_i(t)$ converges to the state emulator $\hat{x}_i(t)$.

Theorem 1. Consider an agent with uncertain dynamics given by (11) and (12), which satisfy condition (6), subject to controller (13), where the feedback gain has been chosen according to (8), with state emulator given by (15), and the adaptive feedback control law given by (16), that exchange local information over a connected, undirected graph $\mathcal{G}$. Then the solution $(\tilde{x}_i(t), B\widetilde{w}_i(t))$ is uniformly exponentially stable for all $(0, B\widetilde{w}_{i0}) \in \mathbb{R}^n \times \mathbb{R}^n$.

Proof. Consider the Lyapunov function candidate for an individual agent given by

$$V(\tilde{x}_i(t), \widetilde{w}_i(t)) = \tilde{x}_i^{\mathrm{T}}(t) P_i \tilde{x}_i(t) + \frac{1}{\alpha} \widetilde{w}_i^{\mathrm{T}}(t) \widetilde{w}_i(t), \quad (23)$$

and note that $V(0,0) = 0$ and $V(\cdot) > 0,\ \forall \tilde{x}_i(t), \widetilde{w}_i(t) \in \mathbb{R} \backslash \{0\}$. Differentiating $V(\cdot)$ along system trajectories (19) and (21) yields

$$\begin{aligned}
\dot{V}(\cdot) &= \dot{\tilde{x}}_i^{\mathrm{T}}(t) P_i \tilde{x}_i(t) + \tilde{x}_i^{\mathrm{T}}(t) P_i \dot{\tilde{x}}_i(t) + \frac{1}{\alpha} \dot{\widetilde{w}}_i^{\mathrm{T}}(t) \widetilde{w}_i(t), \\
&= \tilde{x}_i^{\mathrm{T}}(t) \left[(A - d_i BKC)^{\mathrm{T}} P_i + P_i (A - d_i BKC) \right] \tilde{x}_i(t) \\
&\quad + 2 x_i^{\mathrm{T}}(t) P_i B \widetilde{w}_i(t) - 2 \tilde{x}_i^{\mathrm{T}}(t) P_i B \widetilde{w}_i(t), \\
&= - \tilde{x}_i^{\mathrm{T}}(t) Q_i \tilde{x}_i(t), \\
&\leq - \lambda_{\min}(Q) \| \tilde{x}_i(t) \|_2^2. \quad (24)
\end{aligned}$$

Hence, the closed loop error dynamics given by (19) and (21) are Lyapunov stable for all initial conditions. By evoking the Barbashin-Krasovskii-LaSalle Theorem ([20]), $\tilde{x}_i(t)$ uniformly asymptotically vanishes as $t \to \infty$, and as a result of (19), $B\widetilde{w}_i(t) \to 0$ as $t \to \infty$. Additionally, due to the system's linear time-invariant dynamics, since $(\tilde{x}_i(t), B\widetilde{w}_i(t))$ is uniformly asymptoticly stable, then it is also uniformly exponentially stable ([21]). ∎

Remark 3. Theorem 1 demonstrates that the error system dynamics $(\tilde{x}_i(t), B\widetilde{w}_i(t))$ are exponentially stable, which is sufficient to show that the agent's states, $x_i(t)$, converge to the agent's state emulator, $\hat{x}_i(t)$. However, it is worth noting that, if, in addition to the assumptions outlined in Theorem 1, $B^{\mathrm{T}}B$ is invertible, then it can be shown that the solution $(\tilde{x}_i(t), \widetilde{w}_i(t))$ is uniformly exponentially stable for all $(0, \widetilde{w}_{i0}) \in \mathbb{R}^n \times \mathbb{R}^m$.

Remark 4. Note that Theorem 1 assumes the local agent dynamics satisfy condition (6), and the feedback gain matrix K has been chosen according to (8), which are sufficient conditions for the existence of $P_i \in \mathbb{S}_+$. If a feedback gain K can be found such that solutions to (18) exist, then the results of Theorem 1 hold regardless of the results of Lemma 1.

3.3 Performance and Stability Analysis of the State Emulator

In this section, we rigorously analyze the response of the system state emulator.

To begin, consider the aggregated state vectors given by

$$x(t) = [x_1^{\mathrm{T}}(t), x_2^{\mathrm{T}}(t), \ldots, x_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{nN}, \quad (25)$$

$$\hat{x}(t) = [\hat{x}_1^{\mathrm{T}}(t), \hat{x}_2^{\mathrm{T}}(t), \ldots, \hat{x}_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{nN}, \quad (26)$$

$$\tilde{x}(t) = [\tilde{x}_1^{\mathrm{T}}(t), \tilde{x}_2^{\mathrm{T}}(t), \ldots, \tilde{x}_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{nN}, \quad (27)$$

$$\widetilde{w}(t) = [\widetilde{w}_1^{\mathrm{T}}(t), \widetilde{w}_2^{\mathrm{T}}(t), \ldots, \widetilde{w}_N^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{mN}, \quad (28)$$

$$P = \mathrm{diag}([B^{\mathrm{T}} P_1, \ldots, B^{\mathrm{T}} P_N]) \in \mathbb{R}^{mN \times nN}, \quad (29)$$

and using (16), (19), and (22), the system dynamics can be written in the compact form

$$\dot{\hat{x}}(t) = U \hat{x}(t) + [\mathcal{A}(\mathcal{G}) \otimes BKC] \tilde{x}(t), \quad (30)$$

$$\dot{\tilde{x}}(t) = [\mathrm{I}_N \otimes A - \Delta \otimes BKC] \tilde{x}(t) + [\mathrm{I}_N \otimes B] \widetilde{w}(t), \quad (31)$$

$$\dot{\widetilde{w}}(t) = -\alpha P \tilde{x}(t), \quad (32)$$

with

$$U \triangleq \mathrm{I}_N \otimes A - \mathcal{L}(\mathcal{G}) \otimes BKC. \quad (33)$$

Since we are interested in synchronizing the outputs of all agents, we investigate the properties of the associated graph Laplacian as well as the local agent state transition matrix A. To this end, consider the Jordan decompositions

$$\mathcal{L}(\mathcal{G}) = R J_{\mathcal{L}}(\mathcal{G}) R^{-1}, \quad (34)$$

$$A = S J_A S^{-1}, \quad (35)$$

where R and S are the transformation matrices of the associated graph Lapalcian v and the local agent state transition matrix respectively, and the first column of R is denoted as $p = \mathbf{1}_N$ and the first row of R^{-1} is denoted as q^{T}. Because $\mathcal{L}(\mathcal{G})$ is connected and undirected,

$$\begin{aligned}
J_{\mathcal{L}} &= \begin{bmatrix} \lambda_1 & \ldots & 0 & 0 \\ 0 & \lambda_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \lambda_N \end{bmatrix}, \\
&= \begin{bmatrix} 0 & \mathbf{0}_{N-1}^{\mathrm{T}} \\ \mathbf{0}_{N-1} & \bar{J}_{\mathcal{L}} \end{bmatrix}, \quad (36)
\end{aligned}$$

with λ_i being the i-th eigenvalue of the associated graph Laplacian ordered according to (1), and

$$\bar{J}_{\mathcal{L}} \triangleq \begin{bmatrix} \lambda_2 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \lambda_N \end{bmatrix}. \quad (37)$$

Similarly,

$$J_A = \begin{bmatrix} J_A(0) & \mathbf{0}_{n-r \times r}^{\mathrm{T}} \\ \mathbf{0}_{n-r \times r} & \bar{J}_A \end{bmatrix}, \quad (38)$$

where $J_A(0) \in \mathbb{R}^{r \times r}$ are the aggregated Jordan blocks associated to the zero eigenvalue(s) of the state transition matrix (if A has a non-zero null space), r is the algebraic multiplicity of the zero eigenvalue of A and $\bar{J}_A \in \mathbb{R}^{n-r \times n-r}$ being the Jordan blocks associated with the non-zero eigenvalue(s) of A, which implies $\text{spec}(\bar{J}_A) < 0$. The number of Jordan blocks associated to the zero eigenvalue is given by its geometric multiplicity.

Example 1. To elucidate this point, consider an agent who's dynamics have a zero eigenvalue with geometric multiplicity of 2 and algebraic multiplicity of 3, $J_A(0)$ can be given by

$$J_A(0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

▲

Using the decompositions given in (36) and (38), the state emulator transition matrix U given by (33) can be decomposed as

$$\begin{aligned} J_U =& (R^{-1} \otimes S^{-1})(\mathrm{I}_N \otimes A - \mathcal{L}(\mathcal{G}) \otimes BKC)(R \otimes S), \\ =& (R^{-1}\mathrm{I}_N R \otimes S^{-1}AS) - (R^{-1}\mathcal{L}(\mathcal{G})R \otimes K_S), \\ =& (\mathrm{I}_N \otimes J_A) - (J_{\mathcal{L}} \otimes K_S), \\ =& \begin{bmatrix} J_A & \dots & \mathbf{0}_n & \mathbf{0}_n \\ \mathbf{0}_n & J_A - \lambda_2 K_S & \dots & \mathbf{0}_n \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_n & \mathbf{0}_n & \dots & J_A - \lambda_N K_S \end{bmatrix}, \\ =& \begin{bmatrix} J_A(0) & \mathbf{0}^{\mathrm{T}}_{N(n-r)} \\ \mathbf{0}_{N(n-r)} & \bar{J}_U \end{bmatrix}, \end{aligned} \tag{39}$$

where $K_S = S^{-1}BKCS$, $\bar{J}_U \triangleq \mathrm{I}_{N-1} \otimes J_A - \bar{J}_{\mathcal{L}} \otimes K_S$, λ_k, $k = 1, 2, \dots, N$ are the non-zero eigenvalues of the associated Laplacian, and the system state emulator dynamics can be equivalently given by

$$\dot{\hat{x}}(t) = T^{-1} J_U T \hat{x}(t) + [\mathcal{A}(\mathcal{G}) \otimes BKC]\tilde{x}(t), \tag{40}$$

with $T \triangleq R \otimes S$.

Next, the system given by (40) can be broken into convergent and non-convergent dynamics, where the convergent dynamics exponentially decay to 0, and the non-convergent dynamics are driven to a solution dependent on the local agent dynamics given by A, which will be analyzed in Theorem 3. To this end, let

$$\dot{\hat{z}}(t) = J_A(0)\hat{z}(t) + A_1\tilde{x}(t), \quad \hat{z}(t_0) = \chi_{10}, \tag{41}$$

$$\dot{\hat{c}}(t) = \bar{J}_U \hat{c}(t) + A_2\tilde{x}(t), \quad \hat{c}(t_0) = \chi_{20}, \tag{42}$$

where $\hat{z}(t)$ denotes the non-convergent system dynamics, $\hat{c}(t)$ denotes the convergent system dynamics, $\chi_{10} \in \mathbb{R}^r$ is given by the first r elements of $\hat{x}_0$, $\chi_{20} \in \mathbb{R}^{Nn-r}$ is given by the last $Nn - r$ elements of $\hat{x}_0$, $A_1 \in \mathbb{R}^{r \times Nn}$ is given by the first r rows of $T^{-1}[\mathcal{A}(\mathcal{G}) \otimes BKC]T$, and $A_2 \in \mathbb{R}^{r \times Nn-r}$ is given by the last $Nn - r$ rows of $T^{-1}[\mathcal{A}(\mathcal{G}) \otimes BKC]T$.

Finally, the closed-loop system dynamics given by (31), (32), and (40) can be written in the compact form

$$\dot{\xi}(t) = M\xi(t), \quad \xi(t_0) = [\chi_{10}^{\mathrm{T}}, \chi_{20}^{\mathrm{T}}, 0, \widetilde{w}_0^{\mathrm{T}}]^{\mathrm{T}}, \tag{43}$$

with

$$\xi \triangleq \left[\hat{z}^{\mathrm{T}}(t), \hat{c}^{\mathrm{T}}(t), \tilde{x}^{\mathrm{T}}(t), \widetilde{w}^{\mathrm{T}}(t)\right]^{\mathrm{T}} \in \mathbb{R}^{2Nn+Nm}, \tag{44}$$

and

$$M = \left[\begin{array}{cc|cc} J_A(0) & \mathbf{0} & A_1 & \mathbf{0} \\ \mathbf{0} & \bar{J}_U & A_2 & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathrm{I}_N \otimes A - \Delta \otimes BKC & \mathrm{I}_N \otimes B \\ \mathbf{0} & \mathbf{0} & -\alpha P & \mathbf{0} \end{array}\right], \tag{45}$$

is the partitioned system matrix where the dimensions have been omitted for brevity. The next theorem demonstrates the stability of the closed-loop system dynamics given by (43).

Theorem 2. Consider an agent with uncertain dynamics given by (11) and (12), which satisfy condition (6), subject to controller (13), where the feedback gain has been chosen according to (8), with state emulator given by (15), and the adaptive feedback control law given by (16), that exchange local information over a connected, undirected graph $\mathcal{G}$. Then, the convergent system dynamics given by (42) are exponentially stable.

Proof. Consider the partitioned system transition matrix given by (45), which demonstrates that the convergent and non-convergent dynamics can be decoupled, and the stability of the convergent dynamics given by $\hat{c}(t)$ depend only on

$$\text{spec}(\bar{J}_U) \bigcup \text{spec}\left(\begin{bmatrix} \mathrm{I}_N \otimes A - \Delta \otimes BKC & \mathrm{I}_N \otimes B \\ -\alpha P & 0 \end{bmatrix}\right). \tag{46}$$

Theorem 1 showed that

$$\text{spec}\left(\begin{bmatrix} \mathrm{I}_N \otimes A - \Delta \otimes BKC & \mathrm{I}_N \otimes B \\ -\alpha P & 0 \end{bmatrix}\right) < 0, \tag{47}$$

which implies that we only need demonstrate that $\text{spec}(\bar{J}_U) < 0$. Consider that $\text{spec}(\bar{J}_U)$ is given by

$$\begin{aligned} \text{spec}(\bar{J}_U) =& \left\{\text{spec}\left(J_A - \lambda_k(\mathcal{L}(\mathcal{G}))K_s\right) : k \in [2, N]\right\}, \\ =& \left\{\text{spec}\left(A - \lambda_k(\mathcal{L}(\mathcal{G}))BKC\right) : k \in [2, N]\right\}, \\ <& \ 0, \end{aligned} \tag{48}$$

as direct consequence of Lemma 1, and it follows that the convergent mode system dynamics (42) are exponentially stable. ■

Remark 5. Note that Theorem 2 assumes the local agent dynamics given satisfy condition (6), and the feedback gain K has been chosen according to (8), which are sufficient conditions for $\text{spec}(\bar{J}_U) < 0$. If a feedback gain K can be found such that $\text{spec}(\bar{J}_U) < 0$ holds, then the results of Theorem 2 hold regardless of the results of Lemma 1.

Remark 6. Theorem 2 implies that the states of the local agent system dynamics corresponding to the negative eigenvalues of the local agent system state transition matrix A are exponentially stable and the corresponding shared states of $x_i(t)$ exponentially converge. Note that the stability of the system then depends only on the stability of the states corresponding to the zero eigenvalue(s) of the local agent system state transition matrix A.

In the next theorem, we analyze the stability of the system's non-convergent dynamics given by (41).

Theorem 3. Consider an agent with uncertain dynamics given by (11) and (12), which satisfy condition (6), subject to controller (13), where the feedback gain has been chosen according to (8), with state emulator given by (15), and the adaptive feedback control law given by (16), that exchange local information over a connected, undirected graph $\mathcal{G}$. Then, all agents reach a consensus.

Proof. Consider the non-convergent system dynamics given by (41). Then, the non-convergent state emulator dynamics can be given by

$$\dot{\hat{x}}_r(t) = (p^{\mathrm{T}} \otimes S^{-1})J_A(0)(q \otimes S)\hat{x}_r(t) + A_1\tilde{x}(t), \quad \hat{x}_r(0) = \chi_{10}, \tag{49}$$

with $\hat{x}_r(t)$ denoting the local state emulator states corresponding to zero eigenvalues(s) of the local system transition matrix. Then, the solution to the system described by (49) can be given by

$$\hat{x}_r(t) = (p^{\mathrm{T}} \otimes S^{-1})e^{J_A(0)t}(q \otimes S)\chi_{10} + \int_0^t (p^{\mathrm{T}} \otimes S^{-1})e^{J_A(0)(t-\tau)}(q \otimes S)A_1\tilde{x}(t)d\tau. \tag{50}$$

In Theorem 1, we demonstrated $\tilde{x}(t)$ is uniformly exponentially stable, which implies

$$\int_0^\infty (p^{\mathrm{T}} \otimes S^{-1})e^{J_A(0)(t-\tau)}(q \otimes S)A_1\tilde{x}(t)d\tau = \theta, \tag{51}$$

such that $\|\theta\| < \theta^*$where $\theta \in \mathbb{R}^{r\times 1}$ and θ^* is a computable upper bound. Then, the solution (50) implies that the non-convergent system dynamics can be given by

$$\hat{x}_r(t) \to (\mathbf{1}_N \otimes S)e^{J_A(0)t}(q^{\mathrm{T}} \otimes S^{-1})\hat{x}(t_0) + \theta, \tag{52}$$

and each individual agent's state emulator converges to

$$\hat{x}_i(t) \to \sum_{k\in\mathcal{V}(\mathcal{G})} q_k S e^{J_A(0)t} S^{-1}\hat{x}_k(t_0) + \theta_i. \tag{53}$$

Since θ is bounded, the non-convergent system dynamics are bounded and it follows that all agents reach a consensus as $t \to \infty$. ■

Remark 7. Theorem 3 demonstrates that all agents will reach a consensus as $t \to \infty$ on a quantity determined by the structure $Se^{J_A(0)t}S^{-1}$, which represents the average of each agent's response to the local system dynamics corresponding to the zero eigenvalue(s) of the local state transition matrix A, replicating well known results in literature (see, for example, [22],[23]). In addition, if the system is undisturbed, the non-convergent system dynamics will converge to the quantity given by

$$\hat{x}_i(t) \to \sum_{k\in\mathcal{V}(\mathcal{G})} q_k S e^{J_A(0)t} S^{-1}\hat{x}_k(t_0). \tag{54}$$

Note that (54) may have a non-zero steady state response as demonstrated in the following examples.

Example 2. Consider the case were each agent has first-order integrator dynamics given as $\dot{x}_i = u_i$. In this case, $S = 1$ and $J_A(0) = 0$ and, as a result, $x_i(t) \to q^{\mathrm{T}}x(t_0)$ as $t \to \infty$. Note this example only shows where the states of each agent corresponding to the zero eigenvalue of the state transition matrix A will converge and does not include disturbances. ▲

Example 3. Next, consider the case where local agent dynamics are second-order integrators given as $\ddot{x}_i = u_i$ with the Jordan decomposition of the local state transition matrix A given by $S = \mathrm{I}_2$ and $J_A(0) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. The solution (54) can be given as $\dot{x}_i(t) \to q^{\mathrm{T}}\dot{x}_j(t_0)$, $x_i(t) \to q^{\mathrm{T}}x_j(t_0) + q^{\mathrm{T}}\dot{x}_j(t_0)t$ as $t \to \infty$. Note this example only shows where the states of each agent corresponding to the zero eigenvalues of the state transition matrix A will converge and does not include disturbances. ▲

Remark 8. Note that Theorem 3 assumes the local agent dynamics given satisfy condition (6), and the feedback gain matrix K has been chosen according to (8), which are sufficient conditions for the results of Theorem 1 to hold. If Theorem 1 holds, the results of Theorem 3 hold regardless of the results of Lemma 1.

Remark 9. As shown in equation (50), $\tilde{x}(t)$ acts as a vanishing disturbance to the system's non-convergent dynamics. Note, if $\|\theta\|_2$ is sufficiently small, then agents not only achieve consensus but consensus will occur near the undisturbed system consensus point, $\hat{x}_i(t) \to \sum_{k\in\mathcal{V}(\mathcal{G})} q_k S e^{\theta t} S^{-1}\hat{x}_k(t_0)$. In addition, increasing α will decrease $\|\theta\|_2$. For proof, consider the summation of the Lyapunov function candidates in the proof of Theorem 1 given by

$$V(\tilde{x}(t), \tilde{w}(t)) = \sum_{i\in\mathcal{V}(\mathcal{G})} \tilde{x}_i^{\mathrm{T}}(t)P_i\tilde{x}_i(t) + \tilde{w}_i{}^{\mathrm{T}}(t)\tilde{w}_i(t)/\alpha. \tag{55}$$

Taking the time derivative yields

$$\dot{V}(\tilde{x}(t), \tilde{w}(t)) = -\sum_{i\in\mathcal{V}(\mathcal{G})} \tilde{x}_i^{\mathrm{T}}(t)Q_i\tilde{x}_i(t). \tag{56}$$

Therefore, $V(\tilde{x}(t), \tilde{w}(t)) \leq V(\tilde{x}(t_0), \tilde{w}(t_0)) = \tilde{w}_0^{\mathrm{T}}\tilde{w}_0/\alpha$ and $\sum_{i\in\mathcal{V}(\mathcal{G})} \tilde{x}_i^{\mathrm{T}}(t)P_i\tilde{x}_i(t) \leq \tilde{w}_0^{\mathrm{T}}\tilde{w}_0/\alpha$. As α is increased, the magnitude of the vanishing perturbation term $\|\tilde{x}\|$ becomes smaller, decreasing $\|\theta\|_2$. However, as with all adaptive control architectures, increasing the learning rate excessively may result in reduced time delay margins, highly oscillatory control inputs, and other implementation issues ([24, 25]).

4 Illustrative Numerical Examples

In this section, we demonstrate the efficacy of our approach through several numerical examples. In particular, it is shown that agents achieve consensus in the presence of constant, randomly selected disturbances. In our first example, we consider agents with second order linear dynamics where output feedback is utilized to reach a consensus. In our second example, we reach a consensus on the states of three F-16 aircraft with full state feedback.

4.1 Example 1: Output Feedback

Consider a network of three agents whose dynamics are given by

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; C = [2.99 \quad 1.94], \tag{57}$$

which communicate according to the connected, undirected graph described by

$$\mathcal{L}(\mathcal{G}) = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}. \tag{58}$$

Solving for (7) for this system yields,

$$\bar{P} = \begin{bmatrix} 2.6458 & 1.0000 \\ 1.0000 & 0.6458 \end{bmatrix}, \tag{59}$$

which satisfies (6) and gives $K = 0.3333$. Letting $x_1(0) = [4, -0.5]$, $x_2(0) = [-2, 1]$, and $x_3(0) = [1, -3]$, we see that without control, the states of each agent's dynamics naturally tends to $x_1(\infty) = [3.75, 0]$, $x_2(\infty) = [-1.5, 0]$, and $x_3(\infty) = [-0.5, 0]$. Noting that $q^T = \frac{1}{3}[1\ 1\ 1]$ is a left eigenvalue of (58) and solving (54), we find the undisturbed system approaches $x_{i1}(\infty) = 0.58$ and $x_{i2}(\infty) = 0$ as $t \to \infty$. With only the standard consensus algorithm, Figure 1 shows that the agents cannot reach a consensus in the presence of disturbances, where the constant disturbances are randomly selected as $w_i \in [-1, 1]$. Using the controller in (13) and (16) with the parameters designed above and $\alpha = 1$, the system reaches a consensus even in the presence of constant disturbances as seen in Figure 2. Increasing the learning gain α drives the system closer to the undisturbed system centroid as shown in Figure 3. ▲

4.2 Example 2: F-16 Aircraft

In this example, we consider the longitudinal dynamics of three identical F-16 aircraft whose dynamics are described by

$$A = \begin{bmatrix} -0.0507 & -3.8610 & 0 & -32.2000 \\ -0.0012 & -0.5164 & 0.9283 & -0.0975 \\ -0.0001 & 1.4168 & -2.1382 & -2.2372 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}, \tag{60}$$

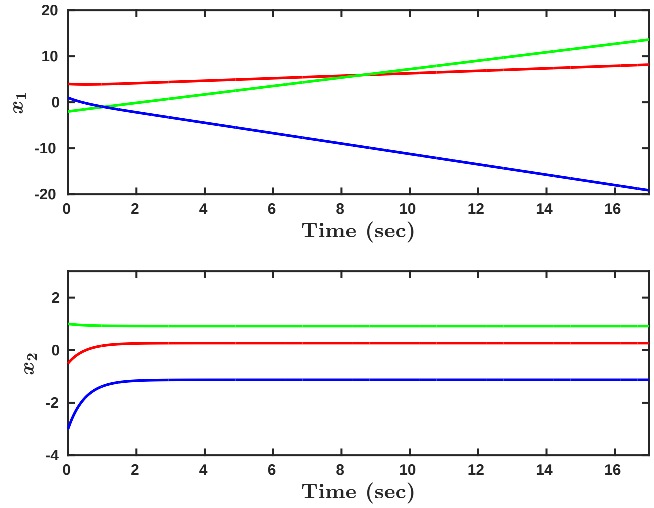

Figure 1. States of each agent subject to constant disturbances with only the standard consensus controller applied.

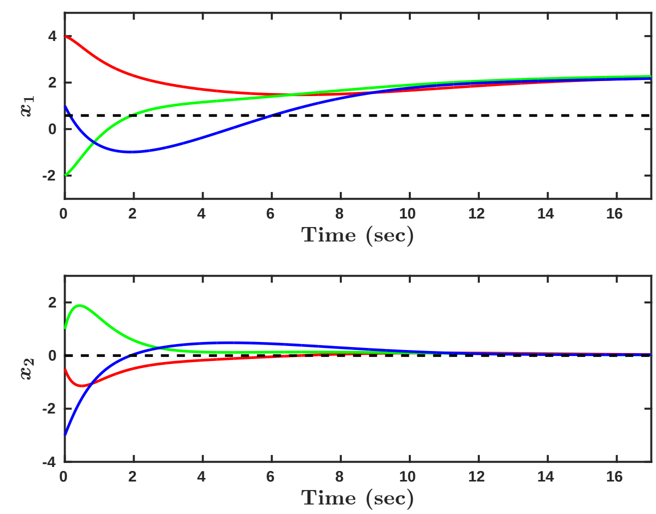

Figure 2. States of each agent with the controller in (13) and (16) applied where $\alpha = 1$. The dashed line indicates the undisturbed system centroid.

$$B = \begin{bmatrix} 0 \\ -0.0717 \\ -1.6450 \\ 0 \end{bmatrix}, \tag{61}$$

as given in [26], where the states μ, α, q, and θ are the *change in aircraft speed*, *angle of attack* (AOA), *pitch rate*, and *pitch*, respectively. Agents exchange their states over a connected, undirected line communication graph. Our aim is to synchronize the aircraft μ values in the presence of disturbances, where the constant disturbances are randomly selected as $w_i \in [-0.08, 0.08]$. Figure 4 demonstrates that the standard consensus controller is in-

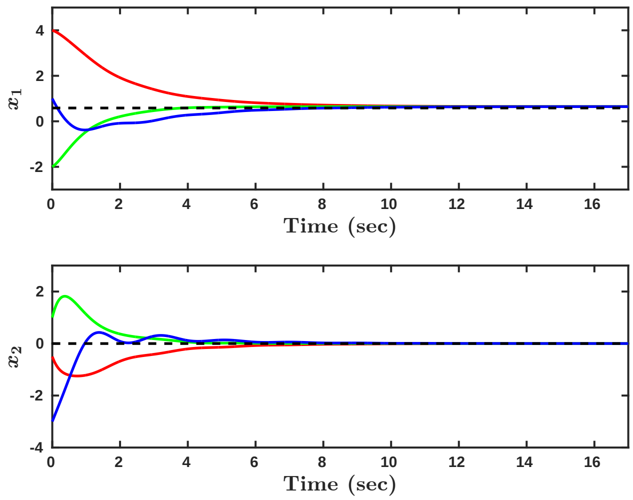

Figure 3. States of each agent with the controller in (13) and (16) applied where $\alpha = 15$. The dashed line indicates the undisturbed system centroid.

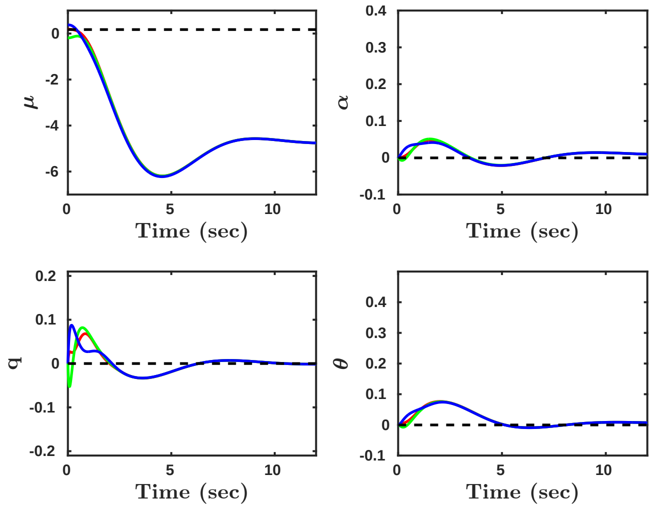

Figure 5. States of each aircraft with the controller in (13) and (16) applied where $\alpha = 1$. The dashed line indicates the undisturbed system centroid.

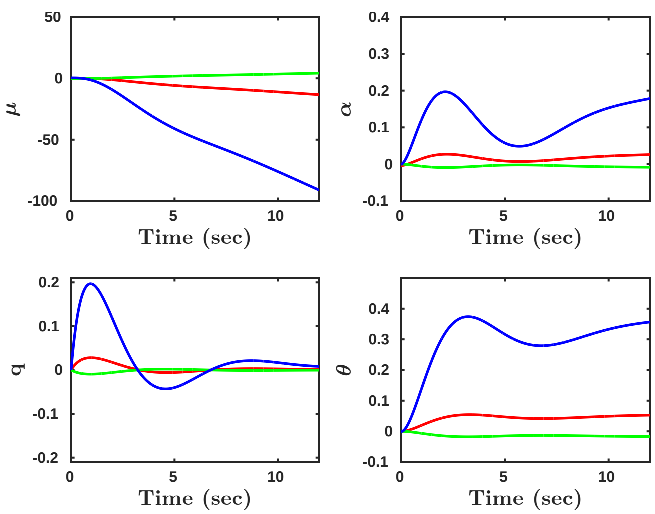

Figure 4. States of each aircraft subject to constant disturbances with only the standard consensus controller applied.

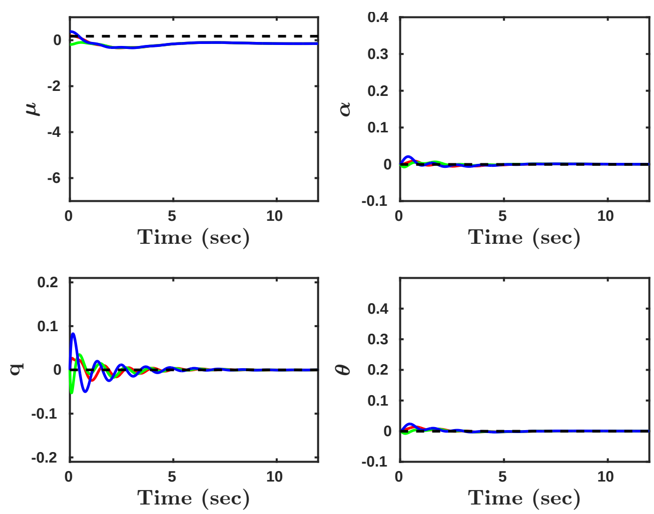

Figure 6. States of each aircraft with the controller in (13) and (16) applied where $\alpha = 15$. The dashed line indicates the undisturbed system centroid.

sufficient to synchronize the system outputs. In Figure 5, we show that using the controller in (13) and (16), the system converges to near the undisturbed system centroid given by (54). Increasing the learning gain α brings the convergence point closer to the undisturbed system centroid as shown in Figure 6. ▲

5 Conclusion

To contribute to resilient networked multiagent control, we have presented a novel state emulator based adaptive control architecture. In particular, we have demonstrated the proposed controller is able to mitigate the effects of constant disturbances and synchronize the outputs of each agent. Unlike previous studies, which make assumptions on agent dynamics and network topologies, the presented results hold for agents with general linear time-invariant dynamics communicating over a connected undirected directed graph, even when all agents are subject to disturbances.

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